

Math 72: 5.7 Solving Polynomial Equations

Objectives

- 1) Solving equations using factoring
- 2) Approximating solutions of equations using GC - intersect
- 3) Approximating solutions of equations using GC - x-intercept ("zero")
- 4) Connect concept of x-intercept of graph with solution of equation

Math 70 Class notes Section 5.8 Solving Equations by Factoring, Problem Solving, and X-intercepts of a Function

Objectives:

- 1) Solve equations by factoring (review Math 45).
- 2) Connect 3.1 (solve equation by GC) to solve equations by factoring.
- 3) Identify a graph by matching algebra for x-intercepts with x-intercepts on graph.
- 4) Solve application problems.

Solve the equation.

$$2x^2 - 10x + 3 = 2x^2 - 10x \quad \text{dist}$$

$$\underline{-2x^2} \quad \underline{-2x^2}$$

$$-10x + 3 = -10x$$

$$\underline{+10x} \quad \underline{+10x}$$

$$3 = 0 \quad \boxed{\text{NO SOLN}}$$

Graph by x-int method

$$y = 2x^2 - 10x - 2x(x-5)$$

Gives horizontal line
 $\Rightarrow$ has no x-ints, no soln

2) $x^3 = 144x$

$$x^3 - 144x = 0$$

set = 0

$$x(x^2 - 144) = 0$$

factor GCF

$$x(x-12)(x+12) = 0 \quad \text{factor diff of sq.}$$

$$\boxed{x = 0, 12, -12}$$

3) $\frac{x^2}{12} + \frac{1}{4} = \frac{x}{3}$ mult by LCD = 12

$$x^2 + 3 = 4x \quad \text{set } = 0$$

$$x^2 - 4x + 3 = 0 \quad \text{factor}$$

$$(x-3)(x-1) = 0$$

$$\boxed{x = 3, 1}$$

Set factors = 0
isolate x.

Graph in GC: $y_1 = x^3$

$$y_2 = 144x$$

WINDOW:

$$x_{\text{MIN}} = -20$$

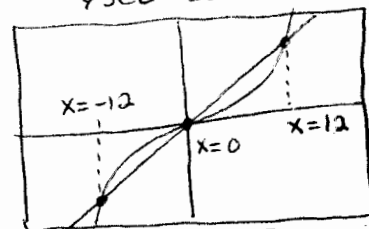
$$x_{\text{MAX}} = 20$$

$$x_{\text{SCL}} = 1$$

$$y_{\text{MIN}} = -2000$$

$$y_{\text{MAX}} = +2000$$

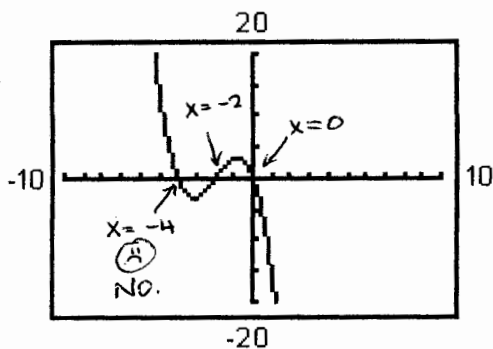
$$y_{\text{SCL}} = 500$$



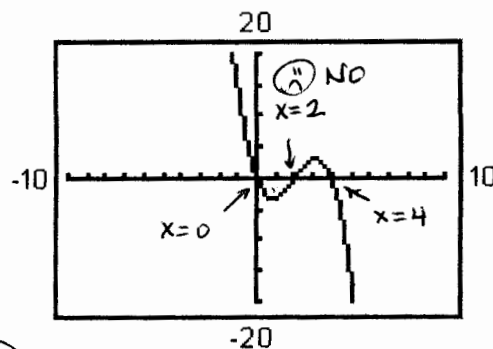
Match the polynomial function with its graph.

4) $f(x) = -x(x+2)(x-4)$ $\leftarrow$ zeros/x-ints at $x = 0, -2, 4$

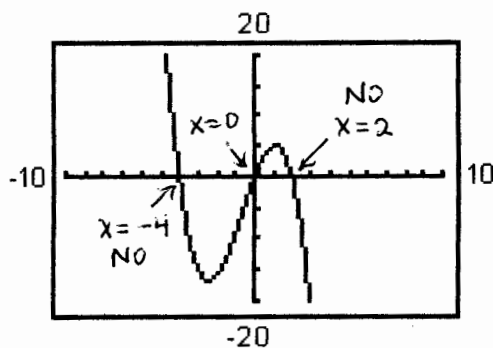
A)



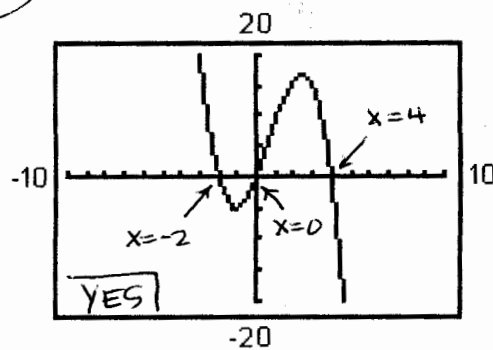
B)



C)



D)



Solve.

- 5) The force in newtons needed to stretch a certain spring x centimeters from its resting position is given by the polynomial function $f(x) = 3x^2$ where 3 is the spring constant. If a force of 48 newtons is applied, how far will the spring be stretched?
- 6) A certain rectangle's length is 5 feet longer than its width. If the area of the rectangle is 66 square feet, find its dimensions.
- 7) Find the length of the shorter leg of a right triangle if the longer leg is 24 meters and the hypotenuse is 6 more than twice the shorter leg.
- 8) A window washer accidentally drops a bucket from the top of a 64-foot building. The height h of the bucket after t seconds is given by $h = -16t^2 + 64$. When will the bucket hit the ground?

EXTRA PRACTICE

Solve the equation.

- 9) $x^3 + 6x^2 = 16x$
- 10) $6x(8x - 3) = 0$
- 11) $9x^2 - 42x + 49 = 0$
- 12) $x(5x + 3)(x - 1) = 0$

Solve.

- 13) One number exceeds another number by 10 and the product of the two numbers is 75. Find the numbers.
- 14) If the cost, $C(x)$, for manufacturing x units of a certain product is given by $C(x) = x^2 - 40x + 9600$, find the number of units manufactured at a cost of \$11,700.
- 15) An object is thrown upward from the top of a 160-foot building with an initial velocity of 48 feet per second. The height h of the object after t seconds is given by the quadratic equation $h = -16t^2 + 48t + 160$. When will the object hit the ground?

M70 5.8 Worksheet solutions (cont)

⑤ Force $f(x) = 3x^2$

to stretch x centimeters.

force 48 Newtons, how far stretched $\Rightarrow$ find x .

$$48 = 3x^2$$

$$16 = x^2$$

$$x = 4 \text{ or } x = -4$$

reject negative distance.
(extraneous)

$$\boxed{4 \text{ cm}}$$

⑥ length = 5 + width.

$$l = 5 + w$$

$$\text{area} = l \cdot w \quad (\text{area of rectangle})$$

$$\text{area} = 66.$$

$$66 = l \cdot w$$

$$66 = (5 + w)w$$

$$66 = 5w + w^2$$

Subst for l to eliminate one variable
distribute

$$0 = w^2 + 5w - 66$$

set = 0, standard form

$$0 = (w - 6)(w + 11)$$

$$\begin{array}{c} -66 \\ -6 \quad 11 \\ 5 \end{array}$$

$$-1, 66$$

$$-2, 33$$

$$-3, 22$$

$$\boxed{-6, 11}$$

factor

$$w - 6 = 0$$

$$w + 11 = 0$$

$$w = 6$$

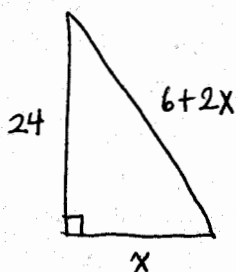
$$w = -11$$

reject negative length
(extraneous)

$$\boxed{w = 6 \text{ ft}}$$

$$l = 5 + w = 5 + 6 = \boxed{11 \text{ ft} = l}$$

⑦ shorter leg = x



Pythagorean Theorem

$$x^2 + 24^2 = (6 + 2x)^2$$

$$x^2 + 576 = 36 + 24x + 4x^2$$

$$0 = \frac{3x^2}{3} + \frac{24x}{3} - \frac{540}{3}$$

$$0 = x^2 + 8x - 180$$

simplify (FOIL)
set = 0.

7) cont.

Factor $x^2 + 8x - 180 = 0$

$(x-10)(x+18) = 0$

$x-10 = 0$

$x+18 = 0$

$x = 10$

~~$x = -18$~~

reject extraneous
(negative length)

$\begin{array}{r} -180 \\ -10 \times 18 \\ 8 \end{array}$

-1, 180

-2, 90

-3, 60

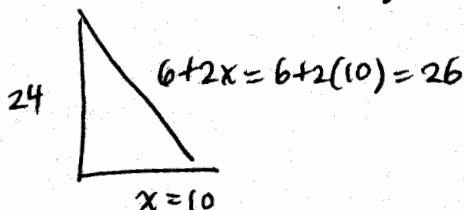
-4, 45

-5, 36

-6, 30

-9, 20

-10, 18



dimensions 10 m, 24 m, 26 m

8) height $h = -16t^2 + 64$

"hit ground" means height $h = 0$.

$0 = -16t^2 + 64$

$\frac{-64}{-16} = \frac{-16t^2}{-16}$

$4 = t^2$

$t = \pm 2$

reject $t = -2$, extraneous
(no negative time)

$t = 2 \text{ sec}$

Solve the equation.

9) $x^3 + 6x^2 = 16x$

$x^3 + 6x^2 - 16x = 0$

$x(x^2 + 6x - 16) = 0$

$x(x+8)(x-2) = 0$

$x = 0$ $x = -8$ $x = 2$

set equal to 0.

factor completely - G.C.F

$\begin{array}{r} -16 \\ 8 \times -2 \\ 6 \end{array}$ trinomial

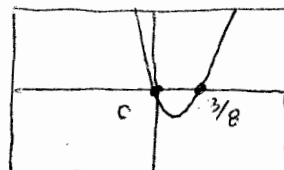
10) $6x(8x-3) = 0$

$6x = 0$ $8x-3 = 0$ set factors = 0 $\rightarrow$

$x = 0$ $x = \frac{3}{8}$

#2 Graph in GC: $y = 6x(8x-3)$

WINDOW: $x_{\min} = -1$, $x_{\max} = 1$



11) $9x^2 - 42x + 49 = 0$

$(3x-7)(3x-7) = 0$

perfect sq.
trinomial.

$3x-7 = 0$

$x = \frac{7}{3}$ double
root (2 identical factors)

M70 5.8 Worksheet solutions (cont)

- (13) $x = \text{one number}$
 $x-10 = \text{another number}$

$x(x-10) = \text{product of the two numbers}$

$$x(x-10) = 75$$

$$x^2 - 10x = 75 \quad \text{distribute}$$

$$x^2 - 10x - 75 = 0 \quad \text{set} = 0, \text{ factor}$$

$$(x-15)(x+5) = 0$$

$$x-15=0 \quad x+5=0$$

$$x=15 \quad x=-5$$

$$\begin{array}{r} -75 \\ 5 \times -15 \\ -10 \end{array}$$

$$\begin{array}{l} 1, -75 \\ 3, -25 \\ 5, -15 \end{array}$$

Nothing says number must be positive!
 Two solutions

$$x=15$$

$$x-10 = 15-10 = 5$$

15 and 5

$$x=-5$$

$$x-10 = -5-10 = -15$$

-15 and -5

- (14) Cost $C(x) = x^2 - 40x + 9600$.

$C = \text{cost}$

$x = \# \text{ units produced.}$

Find # units produced for \$11,700.

Set equal to 11700

$$11700 = x^2 - 40x + 9600$$

$$0 = x^2 - 40x - 2100$$

$$0 = (x+30)(x-70)$$

$$x+30=0 \quad x-70=0$$

$$x = -30 \quad \boxed{x=70}$$

discard negative # of units. (extraneous)

70 units produced at cost \$11700

Set equal to 0.

$$\begin{array}{r} -2100 \\ 30 \times -70 \\ -40 \end{array}$$

factor

$$\begin{array}{l} 1, -2100 \\ 2, -1050 \\ 3, -700 \\ 4, -525 \\ 5, -420 \\ 6, -350 \\ 7, -300 \\ 10, -210 \\ 14, -150 \\ 15, -140 \\ 20, -105 \\ 25, -84 \end{array}$$

30, -70

15

height $h = -16t^2 + 48t + 160$
when will object hit ground $\Rightarrow h = 0$.

$$0 = \frac{-16t^2}{-16} + \frac{48t}{-16} + \frac{160}{-16}$$

$$0 = t^2 - t - 10$$

$$0 = (t-5)(t+2)$$

$$t-5=0 \quad t+2=0$$

$$\boxed{t=5 \text{ sec}}$$

$t=-2$ reject extraneous
neg. time

$$\begin{array}{r} -10 \\ -5 \times 2 \\ -10 \\ \hline -3 \end{array}$$

Math 70: 6.7 Application Problems with Factoring Process

Set up and solve.

1) The product of two consecutive integers is 156. Find the integers.

errors
(1)
(2)

2) The area of a triangle is 320 square inches. The ^{height} ~~leaf~~ is 12 inches longer than it is wide. What are the dimensions of the ~~leaf~~?
triangle?

(3)

3) A valuable sports card is 4 cm wide and 5 cm long. The card is to be sandwiched by two pieces of Lucite, each of which is 5.5 times the ~~area~~ of the card. Determine the dimensions of the Lucite that will ensure a uniform border around the card.

4) The diagonal of a rectangle is 50 feet. The length is 10 feet longer than the width. What are the dimensions of the rectangle?

(4)

5) The force in newtons needed to stretch a certain spring x centimeters from its resting position is given by the polynomial function $f(x) = 3x^2$ where 3 is the spring constant. If a force of ~~38~~ ⁴ newtons is applied, how far will the spring be stretched?

6) A certain rectangle's length is 5 feet longer than its width. If the area of the rectangle is 66 square feet, find its dimensions.

7) Find the length of the shorter leg of a right triangle if the longer leg is 24 meters and the hypotenuse is 6 more than twice the shorter leg.

(5)

8) A window washer accidentally drops a bucket from ^{the} ~~the~~ top of a 64-foot building. The height h of the bucket after t seconds is given by $h(t) = -16t^2 + 64$. When will the bucket hit the ground?

- ① The product of two consecutive integers is 156.
- $\downarrow$ multiply $\downarrow$ two values/expressions $\downarrow$ numbers w/o decimal part in a row $\downarrow$ equal
- $\downarrow$
- x and $(x+1)$

$$x \cdot (x+1) = 156$$

$$x^2 + x = 156$$

distribute

$$x^2 + x - 156 = 0$$

set = 0

$$(x+13)(x-12) = 0$$

$$\begin{array}{r} -156 \\ +13 \quad \times \quad -12 \\ \hline 1 \end{array}$$

$$x = -13 \quad x = 12$$

One pair of consecutive numbers $x = -13$

$$x+1 = -13+1 = -12$$

$$\boxed{-13 \text{ and } -12}$$

Another pair of consecutive numbers $x = 12$

$$x+1 = 12+1 = 13$$

$$\boxed{12 \text{ and } 13}$$

- ② The area of a triangle is 320 in^2 . The height is 12" longer than
- $\downarrow$ $A = \frac{1}{2} B \cdot H$ $\downarrow$ equals $\downarrow$ H $\downarrow$ $=$ $\downarrow$ $12 + B$

$$A = \frac{1}{2} B H = 320$$

... wide. What are the dimensions.

$\downarrow$ B $\downarrow$ B and H

$$\frac{1}{2} B (12+B) = 320$$

mult by $\frac{1}{2}$ to clear fractions

$$B (12+B) = 640$$

dist

$$12B + B^2 = 640$$

set = 0.

$$B^2 + 12B - 640 = 0$$

$$\text{factor } \begin{array}{r} -640 \\ 32 \quad \times \quad -20 \\ \hline 12 \end{array}$$

$$(B+32)(B-20) = 0$$

cont...

Math 70

$$B = -32$$

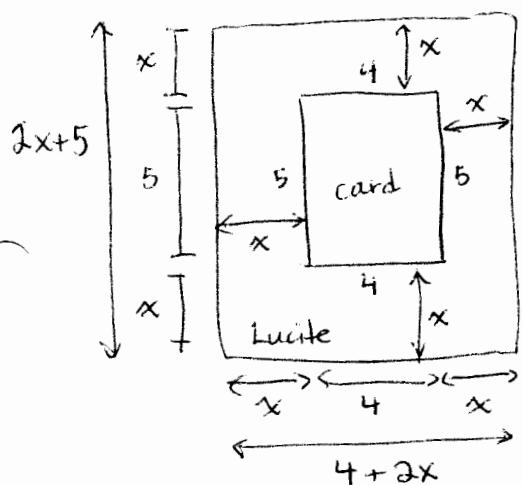
$$B = 20 \text{ in}$$

extraneous
since
lengths
can't be
negative

$$H = B + 12$$

$$H = 20 + 12 = 32 \text{ in} = H$$

- (3) A sports card is 4cm wide and 5cm long, is to be sandwiched between two pieces of Lucite 5.5 times the area of the card. Find dimensions of Lucite that ensure a uniform border



$$\text{Area of card} = L \cdot W = 4 \cdot 5 = 20 \text{ cm}^2$$

$$\text{Area of Lucite} = (5.5)(20) = 110 \text{ cm}^2$$

uniform border $\Rightarrow$ same width x on all four sides.

$$\text{Area of Lucite} = L \cdot W = 110$$

$$(2x+4)(2x+5) = 110$$

subst for L & W.

$$4x^2 + 10x + 8x + 20 = 110$$

FOIL

$$\frac{4x^2}{2} + \frac{18x}{2} - \frac{90}{2} = \frac{0}{2}$$

set = 0

$$2x^2 + 9x - 45 = 0$$

$$2x^2 + 15x - 6x - 45 = 0$$

$$x(2x+15) - 3(2x+15) = 0$$

$$(2x+15)(x-3) = 0$$

$$x = -\frac{15}{2} \text{ extraneous} \quad x = 3 \Rightarrow \text{width of border}$$

$$\begin{array}{r} -90 \\ 15 \times -6 \\ \hline 9 \end{array}$$

1, 90
2, 45
3, 30
5, 18
6, 15

cont...

Math 70

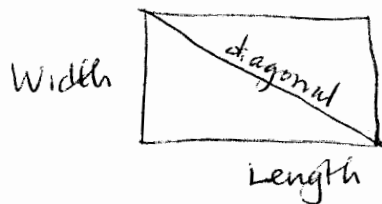
dimensions of Lucite: subst $x=3$

$$W = 4 + 2x \Rightarrow 4 + 2(3) = 4 + 6 = 10$$

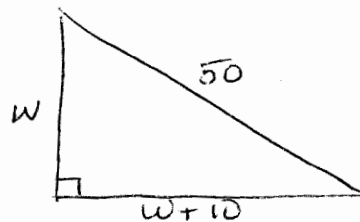
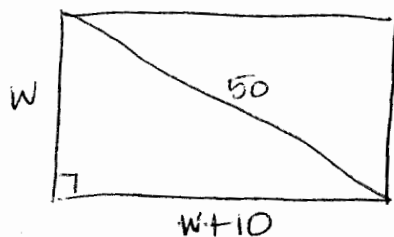
$$L = 5 + 2x \Rightarrow 5 + 2(3) = 5 + 6 = 11$$

dimensions 10 cm x 11 cm

- ④ The diagonal of a rectangle is 50 feet.
↓
equal



The length is 10 ft. longer than width. Find dimensions.
 L equals $10 + W \Rightarrow L = 10 + W$



use right triangle only

Pythagorean Theorem for sides of a right triangle
 $A^2 + B^2 = C^2$

$$W^2 + (W+10)^2 = 50^2$$

substitute

$$W^2 + (W+10)(W+10) = 2500$$

FOIL

$$W^2 + W^2 + 20W + 100 = 2500$$

Set = 0.

$$\frac{2W^2}{2} + \frac{20W}{2} - \frac{2400}{2} = \frac{0}{2}$$

divide all both sides by 2

$$W^2 + 10W - 1200 = 0$$

$$(W-30)(W+40) = 0$$

$$\begin{array}{r} -1200 \\ -30 \times 40 \\ 10 \end{array}$$

$$-10, 120$$

$$-20, 60$$

$$-30, 40$$

$$\downarrow \\ W = 30$$

$$\downarrow \\ W = -40$$

extraneous, lengths can't be negative.

$W = 30 \text{ feet}$

$L = W + 10 = 30 + 10 = 40 \text{ feet} = L$

- (5) Force needed to stretch x cm is $f(x) = 3x^2$. If a force of 48 Newtons is applied, how far stretched? ^{force}
- subst 48 for $f(x)$

$$48 = 3x^2$$

$$\frac{3x^2 - 48}{3} = 0$$

$$x^2 - 16 = 0$$

diff of sq

$$(x-4)(x+4) = 0$$

$$\boxed{x=4 \text{ cm}} \text{ or } x = -4$$

extraneous

- (6) A certain rectangle's length is 5 ft longer than width. If the area is 66 ft^2 , find dimensions.
- $L = 5 + W$

$$L \cdot W = 66$$

area formula

$$L = 5 + W$$

subst

$$(5+W)W = 66$$

dist

$$5W + W^2 = 66$$

set = 0

$$W^2 + 5W - 66 = 0$$

$$(W-6)(W+11) = 0$$

$$\begin{array}{r} -66 \\ -6 \times 11 \\ 5 \end{array}$$

$$W = 6$$

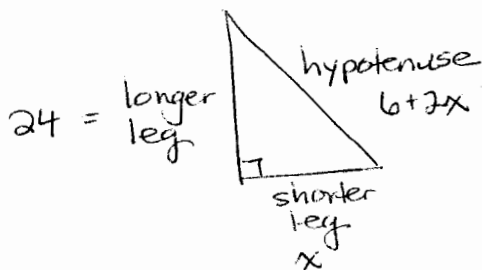
$$W = -11$$

extraneous

$$\boxed{W = 6 \text{ ft}}$$

$$L = W + 5 = 6 + 5 = \boxed{11 \text{ ft} = L}$$

- (7) Find the length of the shorter leg of a right triangle if the longer leg is 24 meters and the hypotenuse is 6 more than twice the shorter leg.



Pythagorean Theorem

$$A^2 + B^2 = C^2$$

$$x^2 + 24^2 = (6+2x)^2$$

$$x^2 + 576 = 36 + 24x + 4x^2$$

$$0 = 3x^2 + 24x - 540$$

cont...

Math 70

$$\frac{3x^2}{3} + \frac{24x}{3} - \frac{540}{3} = \frac{0}{3}$$

$$x^2 + 8x - 180 = 0$$

$$(x+18)(x-10) = 0$$

$$x = -18$$

extraneous

$$\boxed{x = 10 \text{ m}}$$

shorter leg

$$\begin{array}{r} -180 \\ 18 \times -10 \\ \hline 8 \end{array}$$

- ⑧ A window washer accidentally drops a bucket from the top of a 64-foot building. The height h of the bucket after t seconds is given by $h(t) = -16t^2 + 64$. When will the bucket hit the ground?

height = 0
Subst 0 for $h(t)$

$$\frac{0}{-16} = \frac{-16t^2}{-16} + \frac{64}{-16}$$

$$0 = t^2 - 4 \quad \text{diff of sq.}$$

$$(t-2)(t+2)$$

$$\boxed{t = 2}$$

sec

$$t = -2$$

extraneous